Shakura-Sunyaev Disk Can Smoothly Match Advection-Dominated Accretion Flow

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ABSTRACT

We use the standard Runge-Kutta method to solve the set of basic equations describing black hole accretion flows composed of two-temperature plasma. We do not invoke any extra energy transport mechanism such as thermal conduction and do not specify any ad hoc outer boundary condition for the advection-dominated accretion flow (ADAF) solution. We find that in the case of high viscosity and non-zero radiative cooling, the ADAF solution can have an asymptotic approach to the Shakura-Sunyaev disk (SSD) solution, and the SSD-ADAF transition radius is close to the central black hole. Our results further prove the mechanism of thermal instability-triggered SSD-ADAF transition suggested previously by Takeuchi & Mineshige and Gu & Lu.

Subject headings: accretion, accretion disks—black hole physics—hydrodynamics

1. Introduction

The most famous model of accretion disks is the Shakura-Sunyaev disk (SSD; Shakura & Sunyaev 1973). Since SSD was constructed exactly 30 years ago, the most important breakthrough in the field of accretion disk theory has been the proposal of advection-dominated accretion flow (ADAF; Narayan & Yi 1994; Abramowicz et al. 1995). SSD and ADAF appear to be adequate to describe the outer and the inner region of black hole accretion flows, respectively, and a phenomenological SSD+ADAF model has been quite successfully applied to black hole X-ray binaries and galactic nuclei (see Narayan, Mahadevan, & Quataert 1998 for a review). In this model, however, a smooth transition from an outer SSD to an inner ADAF was only assumed, but not proved. From the physical point of view, the question remains whether (and how) an SSD can really match an ADAF.

There have been basically three classes of answers to this question. Dullemond & Turolla (1998) and Molteni, Gerardi, & Valenza (2001) gave negative answers, arguing that a smooth transition from an SSD to an ADAF was not possible. Whereas their conclusions were under

some certain conditions: Dullemond & Turolla (1998) considered only the low-viscosity case (with the viscosity parameter $\alpha \sim 0.1$); and Molteni et al. (2001) referred only to the plain ADAF, i.e. that with zero cooling. The second class of answers, on the other hand, is positive. A number of authors showed that the SSD-ADAF transition was realizable if an extra heat flux caused by thermal conduction was invoked either in the radial direction (Honma 1996; Manmoto & Kato 2000; Gracia et al. 2003), or in the vertical direction (Meyer & Meyer-Hofmeister 1994; Meyer, Liu, & Meyer-Hofmeister 2000). The cost of this class of answers is, in our opinion, the involvement of an additional mechanism of energy transport, and in particular, the introduction of a new unknown parameter α_T to measure thermal conduction (Manmoto & Kato 2000). The third answer was due to Takeuchi & Mineshige (1998) and Gu & Lu (2000), who suggested that the thermal instability of a radiation pressure-supported SSD could trigger the flow to jump from the SSD state to the ADAF state. This answer is also a positive one, but without involving any extra mechanism of energy transport.

In this Letter, we present our answer to the question of SSD-ADAF transition. We demonstrate that such a transition in a smooth way is possible for flows with large values of α (different from the case of Dullemond & Turolla 1998) and non-zero radiative cooling (different from the case of Molteni et al. 2001). We do not involve any extra energy transport mechanism such as conduction, and this is different from the above mentioned second class of answers, and is similar to the third answer. We discuss in some detail the relation between our results here and those of Takeuchi & Mineshige (1998) and Gu & Lu (2000).

2. Equations

The dynamical equations for steady state axisymmetric accretion flows we consider here are usual in the literature (e.g. Narayan, Kato & Honma 1997). That is, the continuity, radial momentum, vertical equilibrium, and angular momentum equations read

$$\dot{M} = -4\pi R H \rho \upsilon \ , \tag{1}$$

$$\upsilon \frac{d\upsilon}{dR} = \Omega^2 R - \Omega_K^2 R - \frac{1}{\rho}\frac{dp}{dR} \ , \tag{2}$$

$$H = \frac{c_s}{\Omega_K} \ , \tag{3}$$

$$\frac{d\Omega}{dR} = \frac{\upsilon \Omega_K (\Omega R^2 - j)}{\alpha R^2 c_s^2} \ , \tag{4}$$

where $\dot{M}$ is the constant mass accretion rate; R is the radius; H is the half-thickness of the flow; ρ is the density of the accreted gas; υ is the radial velocity; Ω is the angular velocity; Ω_K

is the Keplerian angular velocity, and $\Omega_K^2 = GM/(R-R_g)^2 R$ in the well known Paczyński & Wiita (1980) potential, with M being the mass of the central black hole, and R_g being the gravitational radius; p is the pressure; c_s is the isothermal sound speed of the gas, defined as $c_s^2 = p/\rho$; and j is an integration constant that represents the specific angular momentum accreted by the black hole.

As for the energy equation, we employ the form given by Narayan & Yi (1995), which is for flows composed of two-temperature plasma with bremsstrahlung and synchrotron radiation and Comptonization, i.e. a relatively complete and complex case of black hole accretion flows:

$$Q_{vis} = Q_{adv} + Q_{Cou} \ , \tag{5}$$

$$Q_{Cou} = Q_{rad} \ . \tag{6}$$

These two equations are for the energy balance of the ions and of the electrons, respectively. Q_{vis} and Q_{adv} are the rate of viscous heating that is primarily given to the ions and the rate of advective cooling by the ions, and are expressed by, e.g. equations (5) and (6) of Gu & Lu (2000), respectively. Q_{Cou} is the rate of energy transfer from the ions to the electrons through Coulomb collisions, and is expressed by equation (3.3) of Narayan & Yi (1995). Q_{rad} is the rate of radiative cooling of electrons, and is calculated using a bridging formula which is valid in both optically thick and optically thin regimes,

$$Q_{rad} = 8\sigma T_e^4 \left(\frac{3\tau}{2} + \sqrt{3} + \frac{8\sigma T_e^4}{Q_{br} + Q_{sy} + Q_{br,c} + Q_{sy,c}} \right)^{-1} \ , \tag{7}$$

where T_e is the electron temperature; τ is the total (election scattering plus absorption) optical depth, $\tau = \tau_{es} + \tau_{abs} = (0.34 cm^2 g^{-1})\rho H + (Q_{br} + Q_{sy} + Q_{br,c} + Q_{sy,c})/8\sigma T_e^4$; and Q_{br}, Q_{sy}, $Q_{br,c}$, and $Q_{sy,c}$ are the cooling rates of bremsstrahlung radiation, synchrotron radiation, Comptonization of bremsstrahlung radiation, and Comptonization of synchrotron radiation, and are explicitly expressed by equations (3.4), (3.18), (3.23), and (3.24) of Narayan & Yi (1995), respectively.

Finally, the equation of state is needed to close the system of equations,

$$p = p_g + p_r + p_m \ , \tag{8}$$

where $p_g = \Re\rho(T_i + T_e)$ is the gas pressure, T_i is the ion temperature; $p_r = Q_{rad}(\tau + 2/\sqrt{3})/4c$ is the radiation pressure, and $p_m = B^2/8\pi$ is the magnetic pressure, B is the magnetic field, and for simplicity it is usually assumed that $\beta_m \equiv p_m/(p_g + p_m) = \text{const}$.

Note that the dynamical equations (1)-(4) are valid for both geometrically thin and thick flows (Narayan et al. 1997), and equation (7) is a convenient interpolation between

the optically thin and thick limits. When the flow is extremely optically thick, equation (7) gives $Q_{rad} = 16\sigma T_e^4/3\tau$, which is the appropriate black body limit; whereas in the optically thin limit it gives $Q_{rad} = Q_{br} + Q_{sy} + Q_{br,c} + Q_{sy,c}$. Thus we expect that the above set of equations can be used to verify the possible transition from an (optically thick, geometrically thin) SSD to an (optically thin, geometrically thick) ADAF.

3. Numerical Solutions

There are nine equations (equations [1]-[6], [8] plus the definition $p = \rho c_s^2$ and the expression of τ) for nine unknown variables υ, Ω, c_s, H, ρ, p, T_i, T_e and τ as functions of R, with M, $\dot{M}$, α, j, β_m and the adiabatic index γ being constant flow parameters. We use the standard Runge-Kutta method to solve the set of three differential equations (2), (4), and (5) for three unknowns υ, Ω, and c_s, and then obtain other variables from the remaining algebraic equations. We integrate the three differential equations from the sonic point R_s (where the radial velocity is equal to the sound speed) both inward and outward. The derivatives $d\upsilon/dR$, $d\Omega/dR$, and dc_s/dR at R_s, which are needed to start the integration, are evaluated by applying the l'Hôpital rule. The inward, supersonic part of the solution extends to the inner boundary of the flow, i.e. to a radius $R_{\rm in}$ where the no-torque condition $d\Omega/dR = 0$ (i.e. $\Omega R^2 = j$) is satisfied. More important for our purpose here is the outward, subsonic part of the solution. It should be stressed that we do not specify any ad hoc outer boundary conditions. We just observe how the outward solution evolves with increasing R. On the other hand, we obtain a standard SSD solution that is calculated from a set of purely algebraic equations (e.g. Frank, King, & Raine 2002). The given flow parameters M, $\dot{M}$, α and j in the SSD solution are exactly the same as those in the above solution obtained with the Runge-Kutta method. We watch if and where the two solutions can smoothly match with each other.

Figure 1 provides an example of global solution of accretion flow, i.e. the flow quantities as functions of R. The solid line represents the solution of the nine equations in § 2, with given parameters $\alpha = 0.7$, $\dot{m} = 0.01$ ($\dot{m} \equiv \dot{M}/\dot{M}_{\rm Edd}$, with $\dot{M}_{\rm Edd}$ being the Eddington accretion rate), $j = 0.742(cR_g)$, $\gamma = 1.5$, $\beta_m = 0.5$, and $R_s = 2.95R_g$; and the dashed line represents the SSD solution with the same parameters α, $\dot{m}$, and j. Note that R_s is not another free parameter, it is the eigenvalue of the problem, and is self-consistently determined when the constant flow parameters are given. Figure 1(a) is for the radial velocity υ and the sound speed c_s. It is seen that the solid line solution is transonic, with the sonic point being marked by a filled square; while the SSD solution is subsonic everywhere, it alone cannot describe the transonic nature of black hole accretion. Figure 1(b) shows the angular

momentum l ($= \Omega R^2$). The SSD solution follows the Keplerian distribution l_K ($= \Omega_K R^2$), and the solid line solution is sub-Keplerian. Figure 1(c) draws the flow's relative thickness H/R, which is ~ 0.4 (geometrically thick) for small R, decreases as R increases, and reaches to ~ 0.005 (geometrically thin) of the SSD solution. Figure 1(d) is for the optical depth τ, again with increasing R , the flow becomes from being optically thin ($\tau \ll 1$) to being optically thick ($\tau \gg 1$, the SSD solution). Figure 1(e) is for the ion temperature T_i and the electron temperature T_e. The solid line solution has $T_i \gg T_e$; and as R increases, the two temperatures drop down and become identical (the SSD solution). In Figure 1(f) one sees that the advection factor $Q_{adv}/Q_{vis} = (Q_{vis} - Q_{rad})/Q_{vis}$ is ~ 1 (advection-dominated) for small R, and decreases dramatically with increasing R, and reaches nearly zero finally (radiative cooling-dominated, the SSD solution). From these figures we conclude that the solid line solution is an ADAF solution as it has properties of transonic radial motion, sub-Keplerian rotating, and being geometrically thick, optically thin, very hot, and of course, advection-dominated; and that this ADAF solution does match with the (dashed line) SSD solution, forming together a global solution. If the transition radius R_{tr} is defined so that $\tau = 1$ there, then $R_{tr} \approx 12R_g$ in the solution of Figure 1.

4. Discussion

We have shown that a smooth SSD-ADAF transition is realizable for black hole accretion flows with high viscosity ($\alpha = 0.7$ in Figure 1) and non-zero radiative cooling. Our argument is simple and naive. The equations we solve and the numerical method we use are usual. We do not introduce any extra energy transport mechanism such as thermal conduction. Perhaps the only tool somewhat special here is the bridging formula (7) expressing the radiative cooling Q_{rad}, which we need to join the optically thick regime to the optically thin regime of the flow.

Our Figure 1 looks very similar to Figure 1 of Manmoto & Kato (2000), a representative paper of the second class of answers to the question of SSD-ADAF transition as mentioned in Introduction. However, the similar results are obtained in different ways: (1) As mentioned already, Manmoto & Kato (2000) invoked radial thermal conduction and, in particular, introduced a new unknown parameter α_T to measure this extra heat transport mechanism; while we do not. (2) They used the relaxation method to solve the differential equations, and we adopt the Runge-Kutta method. In principle, the solution obtained should not be related to the numerical method, but different methods suit solving different problems. In order to have a solution for the subsonic flow between the sonic point and the outer boundary, the relaxation method requires both the sonic point condition and the

outer boundary condition, and the authors using this method imposed the SSD properties (Keplerian rotating, radiation-dominated, etc.) as the outer boundary condition of ADAF, i.e. the outer boundary of ADAF had been a priori fixed to be in a state corresponding to an SSD. The Runge-Kutta method, on the other hand, requires only one boundary condition, and is adequate for the problem we address here. We do not know a priori what the outer boundary condition of ADAF ought to be, so we do not specify any; but we know for sure that black hole accretion must be transonic, so we use the Runge-Kutta method to integrate the equations starting from the sonic point, and observe how the solution behaves as R increases. For wrong choices of the sonic point condition and the given constant flow parameters, the outward ADAF solution does not match an SSD solution. Then we try again until a correct choice is made and the ADAF solution has an asymptotic approach to an SSD solution that corresponds to the same flow parameters as those for the ADAF solution, thus we believe that a global solution containing an SSD-ADAF transition is found. The payment for using the Runge-Kutta method is that not only the variables at the sonic point, but also their derivatives there must be supplied in order to start the integration, the calculations applying the l'Hôpital rule are troublesome, though still straightforward. (3) In Manmoto & Kato (2000) the transition radius R_{tr} was an inputted free parameter, while in our work it is determined by the constant flow parameters and is naturally calculated.

Let us now comment on the relation between our results here and the third answer to the question of SSD-ADAF transition mentioned in Introduction. Takeuchi & Mineshige (1998) suggested firstly that for large $\alpha \sim 1$ the thermal instability of radiation pressure-supported SSD could trigger the SSD-ADAF transition. They made time evolutionary calculations and obtained very interesting results: because of the dominance of radiation pressure, the SSD becomes unstable at a radius ($\sim 3.5R_g$ in their example solution, see their Figures 2 and 3), and the outer stable parts of the flow are disturbed and evolve towards the ADAF state; this outward propagating disturbance damps and stops at a large radius ($\sim 160R_g$), then a transition backward to the SSD state starts from that radius, and propagates inward until $\sim 5R_g$ (it is larger than the original instability radius $\sim 3.5R_g$); finally, a two-phased flow structure really becomes persistent, i.e. the flow stays in the ADAF state inside the transition radius $R_{tr} \sim 5R_g$, and in the SSD state outside it. Later, Gu & Lu (2000) made a somewhat more extensive study on the mechanism of thermal instability-triggered SSD-ADAF transition, giving an $\alpha - \dot{m}$ parameter diagram in which the region allowing the SSD-ADAF transition is clearly seen. Both Gu & Lu (2000) and the present paper are for stationary flows, so if the work of Gu & Lu (2000) corresponds to the first step of time evolutionary calculations of Takeuchi & Mineshige (1998), i.e. it proves the cause of SSD-ADAF transition and shows how to determine the original instability radius where the transition starts to occur, then our work here corresponds to the final stage of Takeuchi &

Mineshige's evolutionary sequence, i.e. it demonstrates that a stable two-phased flow can form and exist.

In order to see more clearly the relation between the original instability radius R_b (where the SSD solution starts to break off due to the thermal instability) and the transition radius R_{tr} (where the SSD solution matches the ADAF solution in the final two-phased structure), we show in Figure 2 how these two radii vary with $\dot{m}$. In this figure the solid line for R_{tr} is obtained by numerically solving the set of equations listed in § 2, with $\gamma = 1.5$, $\beta_m = 0.5$, $\alpha = 0.7$, and $j = 0.742(cR_g)$ (then for each value of $\dot{m}$ a correctly chosen value of R_s is required in order to obtain a solution that contains an SSD-ADAF transition); and the dashed line for R_b is drawn by applying the instability condition $\beta \equiv p_g/(p_g + p_r) = 0.4$ in the standard SSD theory, which gives $R_b \propto \dot{m}^{16/21}$ (e.g. Kato, Fukue & Mineshige 1998). The solution of Figure 1 corresponding to $\dot{m} = 0.01$ is marked by filled squares, which has $R_{tr} \approx 12R_g$ and $R_b \approx 4.5R_g$. It is seen that R_{tr} is insensitive to $\dot{m}$, and is always larger than R_b; and the larger $\dot{m}$ is, the closer the two radii are. It is also clear from Figure 2 that R_{tr} is close to the central black hole, and this is because, according to Takeuchi & Mineshige (1998) and Gu & Lu (2000), the SSD-ADAF transition is caused by the thermal instability in the radiation pressure-supported region, i.e. in the very inner part of SSD. These results about R_{tr} are distinctive from those in other transition mechanisms. For example, in the SSD-ADAF transition model involving radial thermal conduction, R_{tr} appears as an inputted free parameter and has a very wide range, i.e. from a few to $\sim 10^4 R_g$ (Manmoto & Kato 2000). It is worth studying further whether the SSD-ADAF transition radius ought to be close to the central black hole or it could be far away from the hole.

This work was supported by the National Science Foundation of China under Grant No. 10233030.

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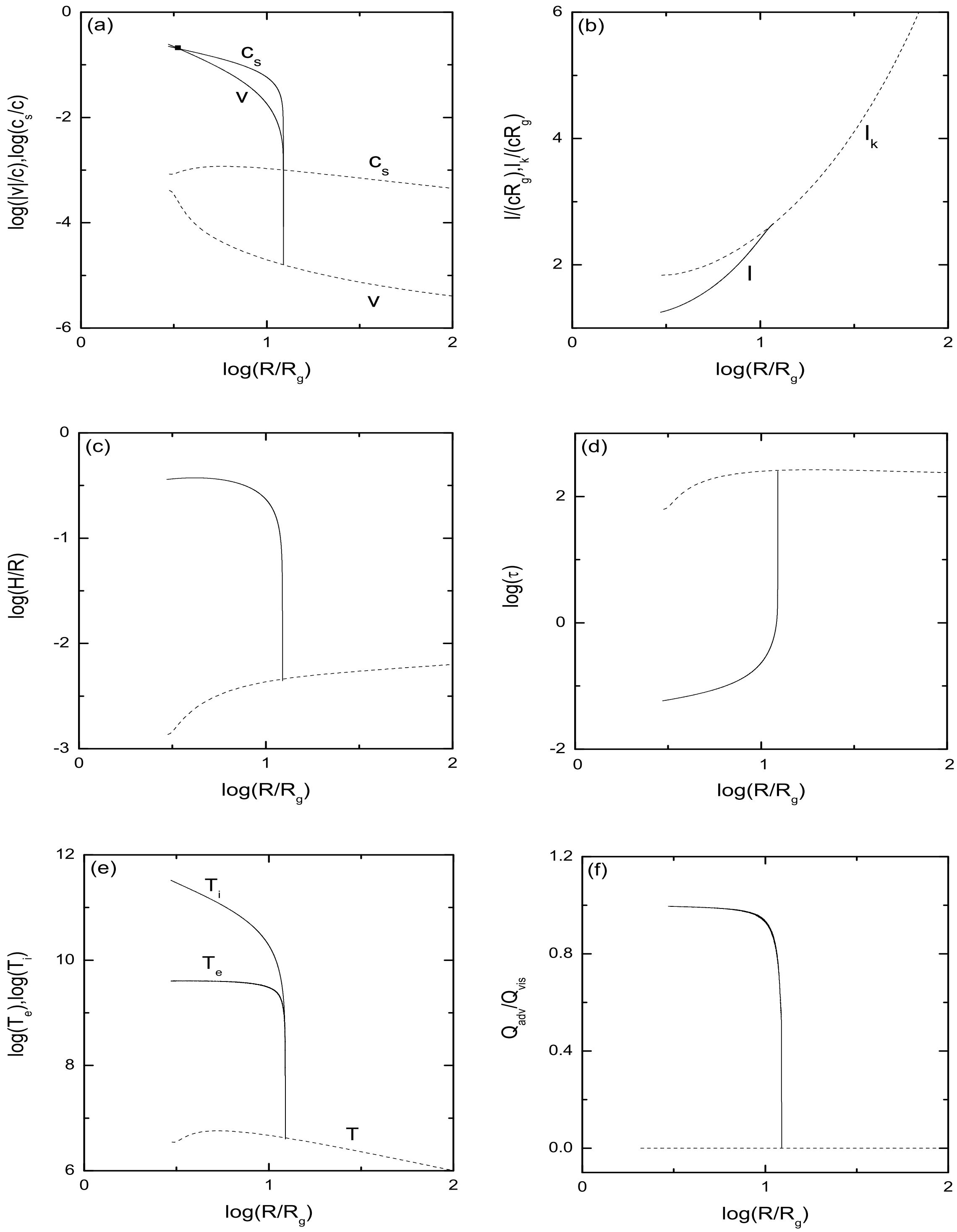

Fig. 1.— A global solution containing an SSD-ADAF transition. The solid line and the dashed line represent the ADAF solution and the SSD solution, respectively. (a), (b), (c), (d), (e), and (f) are for the radial velocity v and the sound speed c_s, the angular momentum l and the Keplerian angular momentum l_K, the relative thickness H/R, the optical depth τ, the ion temperature T_i and the electron temperature T_e, and the advection factor Q_{adv}/Q_{vis}, respectively.

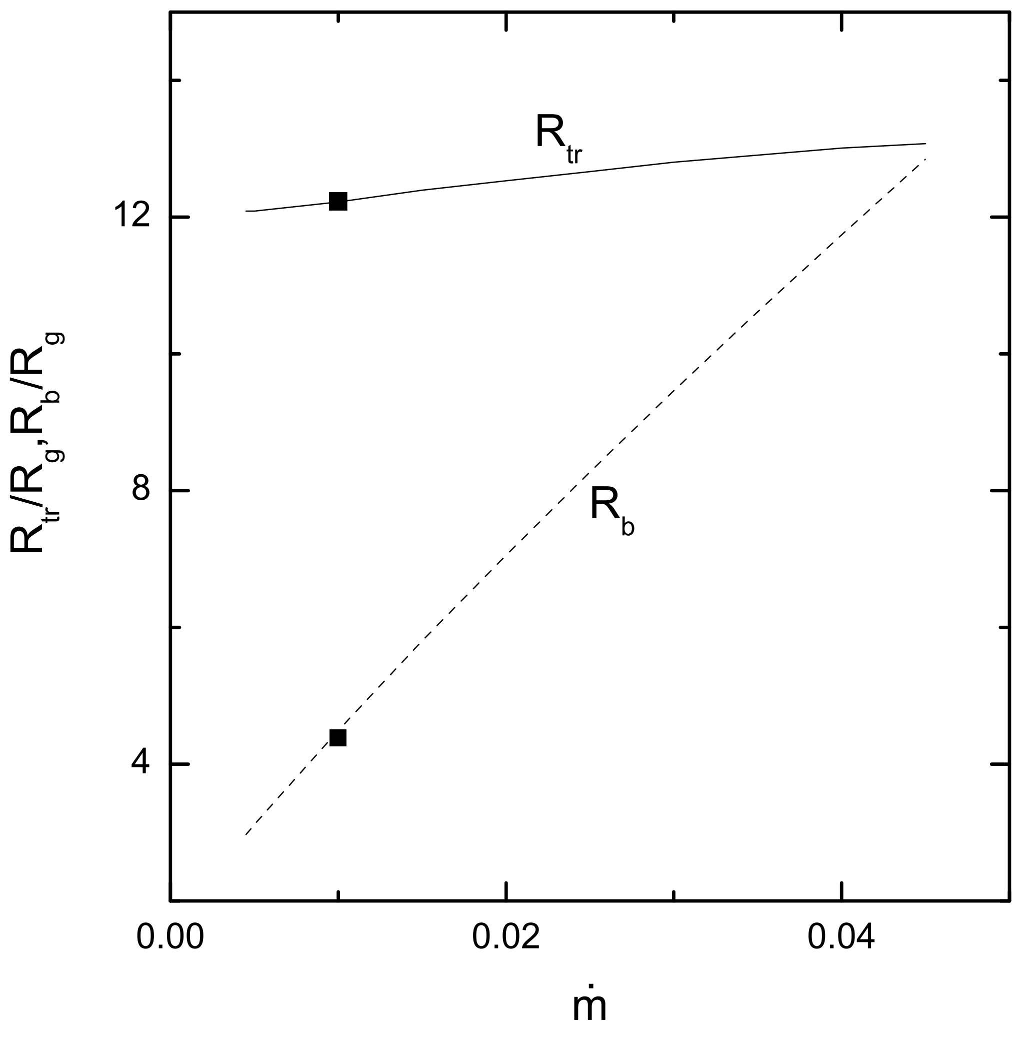

Fig. 2.— Dependences of the SSD-ADAF transition radius R_{tr} and the thermal instability radius R_b on the accretion rate $\dot{m}$.